NAG Toolbox for MATLAB

g07ab

1 Purpose

g07ab computes a confidence interval for the mean parameter of the Poisson distribution.

2 Syntax

```
[tl, tu, ifail] = g07ab(n, xmean, clevel)
```

3 Description

Given a random sample of size n, denoted by $x_1, x_2, \ldots, x_n$, from a Poisson distribution with probability function

$$p(x) = e^{-\theta}\frac{\theta^x}{x!}, \qquad x = 0, 1, 2, \ldots$$

the point estimate, $\hat{\theta}$, for θ is the sample mean, $\bar{x}$.

Given n and $\bar{x}$ this function computes a $100(1-\alpha)\%$ confidence interval for the parameter θ, denoted by $[\theta_l, \theta_u]$, where α is in the interval $(0, 1)$.

The lower and upper confidence limits are estimated by the solutions to the equations

$$e^{-n\theta_l}\sum_{x=T}^{\infty}\frac{(n\theta_l)^x}{x!} = \frac{\alpha}{2},$$

$$e^{-n\theta_u}\sum_{x=0}^{T}\frac{(n\theta_u)^x}{x!} = \frac{\alpha}{2},$$

where $T = \sum_{i=1}^{n} x_i = n\hat{\theta}$.

The relationship between the Poisson distribution and the χ^2-distribution (see page 112 of Hastings and Peacock 1975) is used to derive the equations

$$\theta_l = \frac{1}{2n}\chi^2_{2T,\alpha/2},$$
$$\theta_u = \frac{1}{2n}\chi^2_{2T+2,1-\alpha/2},$$

where $\chi^2_{\nu,p}$ is the deviate associated with the lower tail probability p of the χ^2-distribution with ν degrees of freedom.

In turn the relationship between the χ^2-distribution and the gamma distribution (see page 70 of Hastings and Peacock 1975) yields the following equivalent equations;

$$\theta_l = \frac{1}{2n}\gamma_{T,2;\alpha/2},$$
$$\theta_u = \frac{1}{2n}\gamma_{T+1,2;1-\alpha/2},$$

where $\gamma_{\alpha,\beta;\delta}$ is the deviate associated with the lower tail probability, δ, of the gamma distribution with shape parameter α and scale parameter β. These deviates are computed using g01ff.

4 References

Hastings N A J and Peacock J B 1975 *Statistical Distributions* Butterworths

Snedecor G W and Cochran W G 1967 *Statistical Methods* Iowa State University Press

5 Parameters

5.1 Compulsory Input Parameters

1: **n – int32 scalar**

n, the sample size.

Constraint: $\mathbf{n} \geq 1$.

2: **xmean – double scalar**

The sample mean, $\bar{x}$.

Constraint: $\mathbf{xmean} \geq 0.0$.

3: **clevel – double scalar**

The confidence level, $(1 - \alpha)$, for two-sided interval estimate. For example $\mathbf{clevel} = 0.95$ gives a 95% confidence interval.

Constraint: $0.0 < \mathbf{clevel} < 1.0$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **tl – double scalar**

The lower limit, θ_l, of the confidence interval.

2: **tu – double scalar**

The upper limit, θ_u, of the confidence interval.

3: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

$\mathbf{ifail} = 1$

On entry, $\mathbf{n} < 1$,
or $\mathbf{xmean} < 0.0$,
or $\mathbf{clevel} \leq 0.0$,
or $\mathbf{clevel} \geq 1.0$.

ifail $= 2$

When using the relationship with the gamma distribution to calculate one of the confidence limits, the series to calculate the gamma probabilities has failed to converge. Both **tl** and **tu** are set to zero. This is a very unlikely error exit and if it occurs please contact NAG.

7 Accuracy

For most cases the results should have a relative accuracy of $\max(0.5\mathrm{D}{-}12, 50.0 \times \epsilon)$ where ϵ is the ***machine precision*** (see x02aj). Thus on machines with sufficiently high precision the results should be accurate to 12 significant digits. Some accuracy may be lost when $\alpha/2$ or $1 - \alpha/2$ is very close to 0.0, which will occur if **clevel** is very close to 1.0. This should not affect the usual confidence intervals used.

8 Further Comments

None.

9 Example

```
n = int32(98);
xmean = 3.020408163265306;
clevel = 0.95;
[tl, tu, ifail] = g07ab(n, xmean, clevel)
```

```
tl =
    2.6861
tu =
    3.3848
ifail =
           0
```